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applied to the last result, give after simplifications,

$$\{(M+m)R^2 + Mk'^2\}\ddot{\varphi} - mRr'\cos(\theta - \alpha)\ddot{\theta} + mRr'\sin(\theta - \alpha)\dot{\theta}^2 = g(M+m)R\sin\alpha,$$

$$Rr'\cos(\theta - \alpha)\ddot{\varphi} - (r'^2 + k'^2)\ddot{\theta} = gr'\sin\theta.$$

Eliminating  $\ddot{\varphi}$ ,

$$[(r'^2 + k'^2)\{(M+m)R^2 + Mk'^2\} + mR^2r'^2\cos^2(\theta - \alpha)]\ddot{\theta} - mR^2r'^2\sin(\theta - \alpha)\cos(\theta - \alpha)\dot{\theta}^2$$

$$= -gr'[\{(M+m)R^2 + Mk'^2\}\sin\theta + (M+m)R^2\sin\alpha\cos(\theta - \alpha)].$$

Multiplying by  $2\dot{\theta}$  and integrating

$$[(r'^2 + k'^2)\{(M+m)R^2 + Mk'^2\} + mr'^2R^2\cos^2(\theta - \alpha)]\dot{\theta}^2$$

$$= gr'[2\{(M+m)R^2 + Mk'^2\}\cos\theta - 2(M+m)R^2\sin\alpha\sin(\theta - \alpha)] + C',$$

which is of the same general form as (7), p. 351, this MONTHLY for November, 1916.

### NUMBER THEORY.

#### 235. Proposed by W. D. CAIRNS, Oberlin College.

Prove that  $n = 1$  is the only positive integer for which  $n^4 + 4$  is a prime.

SOLUTION BY WM. E. PATTEN, Government Institute of Technology, Shanghai, China.

$$n^4 + 4 = (n^4 + 4n^2 + 4) - 4n^2 = (n^2 + 2)^2 - (2n)^2 = (n^2 + 2n + 2)(n^2 - 2n + 2).$$

Therefore,  $n^4 + 4$  is a prime, if at all, only for those values of  $n$  which make either  $n^2 + 2n + 2 = 1$ , or  $n^2 - 2n + 2 = 1$ , since each of the factors of  $n^4 + 4$  given above is integral in value when  $n$  is integral, and both are positive when  $n$  is positive.

(1) When  $n^2 + 2n + 2 = 1$ , then  $n = -1$ .

(2) When  $n^2 - 2n + 2 = 1$ , then  $n = +1$ . When  $n = +1$ , then  $n^4 + 4 = 5$ , a prime.

Therefore,  $n^4 + 4$  is a prime for  $n = 1$ , and for no other positive integral values of  $n$ .

Also solved by ELMER SCHUYLER, FRANK IRWIN, HORACE OLSON, ELIJAH SWIFT, H. H. CLARK, ELIZABETH B. DAVIS, NORMAN ANNING, L. G. WELD, and the PROPOSER.

#### 236. Proposed by V. M. SPUNAR, Chicago, Illinois.

Find integral values of  $x, y, z$ , such that

$$xy + z = \square, \quad yz + x = \square, \quad \text{and} \quad xz + y = \square.$$

SOLUTION BY ARTEMAS MARTIN, LL.D., Washington, D. C.

Assume  $x = n^2, y = (n+1)^2$ ; then the given equation becomes

$$n^2(n+1)^2 + z = \square, \quad (n+1)^2z + n^2 = \square, \quad n^2z + (n+1)^2 = \square.$$

Put

$$n^2(n+1)^2 + z = a^2.$$

Assume  $a = n^2 + n + b$ , and the last equation becomes

$$n^2(n+1)^2 + z = a^2 = (n^2 + n + b)^2,$$

from which we immediately find

$$z = b(2n^2 + 2n + b).$$

Substituting in

$$(n+1)^2z + n^2 = \square,$$

we have

$$b(n+1)^2(2n^2 + 2n + b) + n^2 = \square = c^2,$$

which is satisfied by  $b = 2$ ; for then

$$c^2 = 4n^4 + 12n^3 + 17n^2 + 12n + 4 = (2n^2 + 3n + 2)^2,$$

and  $z = 4(n^2 + n + 1)$ . This value of  $z$ , with the assumed values,  $x = n^2$ ,  $y = (n + 1)^2$ , satisfies all the proposed conditions.

$$xy + z = n^2(n + 1)^2 + 4(n^2 + n + 1) = (n^2 + n + 2)^2,$$

$$yz + x = 4(n + 1)^2(n^2 + n + 1) + n^2 = (2n^2 + 3n + 2)^2,$$

$$xz + y = 4n^2(n^2 + n + 1) + (n + 1)^2 = (2n^2 + n + 1)^2.$$

If  $n = 1$ , then  $x = 1$ ,  $y = 4$ ,  $z = 12$ .

If  $n = 2$ , then  $x = 4$ ,  $y = 9$ ,  $z = 28$ .

If  $n = 3$ , then  $x = 9$ ,  $y = 16$ ,  $z = 52$ .

And so on, indefinitely.

The values of  $x$ ,  $y$ ,  $z$  just found will also satisfy the conditions

$$xy + x + y = \square, \quad xz + x + z = \square, \quad \text{and} \quad yz + y + z = \square.$$

Also solved by ELIZABETH B. DAVIS and H. N. CARLETON.

**237. Proposed by NORMAN ANNING, Chilliwack, B. C.**

Prove that for three numbers  $x$ ,  $y$ ,  $z$ ,

$$9\Sigma(x - y)^4 = \Sigma(2x - y - z)^4 = 2\square.$$

SOLUTION BY E. F. CANADAY, University of South Dakota.

This problem is evidently misprinted. If we write it

$$9\Sigma(x - y)^4 = \Sigma(2x - y - z)^4 = 2\square,$$

a solution is possible. To prove

$$9[(x - y)^4 + (y - z)^4 + (z - x)^4] = (2x - y - z)^4 + (2y - z - x)^4 + (2z - x - y)^4 = 2\square,$$

we put

$$(x - y) = a, \quad (y - z) = b, \quad \text{and} \quad (z - x) = -(a + b).$$

Then

$$\begin{aligned} 9[a^4 + b^4 + (-a - b)^4] &= (2a + b)^4 + (b - a)^4 + (-a - 2b)^4 = 9(2a^4 + 4a^3b + 6a^2b^2 \\ &\quad - 4ab^3 + 2b^4) = 16a^4 + 32a^3b + 24a^2b^2 + 8ab^3 + b^4 + b^4 - 4ab^3 + 6a^2b^2 - 4a^3b + a^4 + a^4 \\ &\quad + 8a^3b + 24a^2b^2 + 32ab^3 + 16b^4 = 2[9(a^4 + 2a^3b + 3a^2b^2 + 2ab^3 + b^4)] = 18a^4 + 36a^3b \\ &\quad + 54a^2b^2 + 36ab^3 + 18b^4 = 2\square. \end{aligned}$$

$$2[3(a^2 + ab + b^2)]^2 = 2\square.$$

Also solved by the PROPOSER.

## QUESTIONS AND DISCUSSIONS.

SEND ALL COMMUNICATIONS TO U. G. MITCHELL, University of Kansas, Lawrence, Kansas.

### REPLIES.

**20.** Some of our readers would like to have a simple account, without proofs, of just what has been accomplished toward the proof of the theorem that the equation  $x^n + y^n = z^n$  is impossible in integers when  $n > 2$ .

Readers interested in the above question will be glad to learn that a better and more complete article than that contemplated as an answer to the question